

Alluring Anti-diagonal - Solution

Using Laplace expansion of rows, we know that the determinant equals the product of the diagonal elements and then maybe multiplied by -1. Note that if n is even, we have to multiply by -1 when doing Laplace expansion on the first column. If n is odd, we do not have to multiply by -1 when doing the expansion of the first column. Then for expanding the second column, we have to multiply by -1 if $n - 1$ is even. In total for these two columns, we see that we have to multiply by -1 if $n + n - 1$ is odd. Repeating this procedure for all columns, we see that we have to multiply by -1 if

$$n + (n - 1) + \dots + 1$$

is odd, with the exception that if $n = 1$, we don't. Note that

$$n + (n - 1) + \dots + 1 = \sum_{i=1}^n i = n(n + 1)/2.$$

So we get that the determinant equals

$$b_i \prod_i a_i$$

where b_i equals -1 if n is not 1 and $n(n + 1)/2$ is odd.